Free Vibrations of Rocket Motor Casings under Internal Pressure

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Theme

THE dynamic behavior of shells of revolution under initial stress is of importance in the structural design of space vehicles. Previous investigations in this area are mainly confined to cylindrical shells,[1-3] though some attention is given to spherical and conical shells.[4] Yang et al.[5] have applied a finite element method to predict the vibration characteristics of cylindrical and parabolic shells under linearly varying initial stress. This paper deals with the vibrations of a simply supported cylindrical shell with torispherical heads subjected to uniform internal pressure. The effects of geometric parameters on frequencies are studied. It is observed that the meridional rotations, caused by internal pressure, affect the frequencies of short shells.

Contents

The governing differential equations for initially stressed and vibration states are derived using Sanders' nonlinear thin shell theory.[6] A linear bending analysis is performed to evaluate the axisymmetric, initially stressed state. A set of linear homogeneous equations of vibration is obtained by perturbing this stressed state. All these equations, written in first-order form are solved using a segmentation technique.[7] The integration of differential equations, corresponding to vibration state, with homogeneous boundary conditions leads to a nonvanishing determinant. The characteristic equation of this determinant is a cubic in Ω^2 ($\Omega = \omega/\omega_s$ where $w_s = \pi/t\,[E/2(1+\nu)\rho]^{1/2}$). This yields three frequencies of physical significance for an assumed nodal pattern. These frequencies refer to the longitudinal, circumferential and transverse modes of vibration. It is generally observed that the lowest frequency of a shell corresponds to a flexural mode of vibration, while the other two frequencies are very much higher in magnitude. Hence the fundamental flexural frequencies are of interest to a design engineer so as to avoid any catastrophic failure of a structure.

Numerical Results

The geometry of the shell is shown in Fig. 1. The parameters r_0/R and R_t/R are taken as 0.2 and 0.25, respectively. The material properties of the shell are: $E = 2.1 \times 10^6$ Kg/cm^2; $\nu = 0.3$, and $\rho = 7.8$ gm/cm^3. The boundary conditions for the initially stressed state are: $\bar{W} = \bar{M}_s = \bar{N}_s = 0$ where $\bar{W} = W/t$; $\bar{M}_s = M_s/Et^2$; $\bar{N}_s = N_s/Et$.

Severe bending stresses occur in the torodial portion of the shell because of a sudden change in curvature between the spherical and toroidal segments. The meridional and circumferential stress resultants $\bar{N}_s$ and $\bar{N}_\theta (= N_\theta/Et)$ along the shell length are shown in Fig. 1. It is seen that $\bar{N}_s$ is almost constant except near the edge where the $\bar{N}_s = 0$ condition is imposed. The stress resultant $\bar{N}_\theta$ is compressive in the toroidal zone and tensile over the cylindrical portion.

Flexural frequencies of a torispherical shell corresponding to a half wave in the longitudinal direction are presented in Figs. 2-4. The effect of internal pressure on the square of frequency parameter Ω, for different circumferential waves (n), is shown in Fig. 2. For a particular n, the square of frequency linearly varies with the internal pressure $\bar{q}(\bar{q} = q/E)$. Because of the stiffening effect of internal pressure, the wave number corresponding to the fundamental frequency is small as $\bar{q}$ increases.

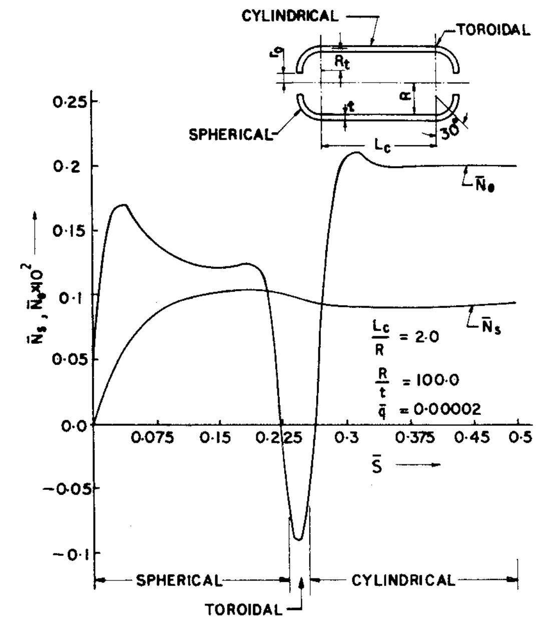

Fig. 1 Variation of initial stress resultants along the shell length.

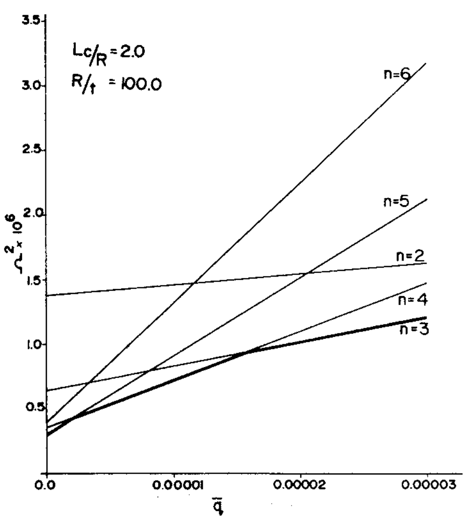

Fig. 2 Variation of frequency with $\bar{q}$.

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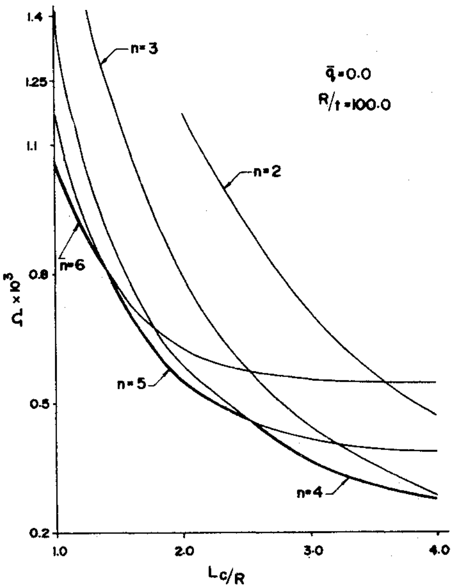

Fig. 3 Variation of frequency with L_c/R ($\bar{q}=0.0$).

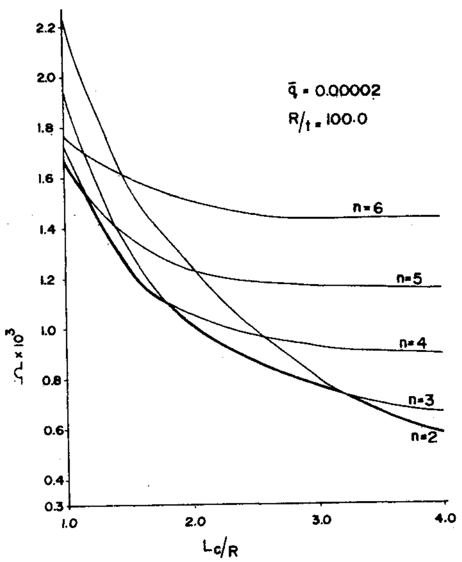

Fig. 4 Variation of frequency with L_c/R ($\bar{q}=0.00002$).

Figures 3 and 4 represent the variation of frequency parameter Ω, with L_c/R ratios (where L_c and R are length and radius of the cylinder, and t is the thickness of the shell) for unstressed and stressed cases respectively. The parameter Ω, for an assumed n, reduces as L_c/R increases and tends to a constant value for high L_c/R ratios. The fundamental frequency and the associated wave number n, decrease for increasing L_c/R ratios. Further, the circumferential wave number n, at the fundamental frequency, for a stressed shell is lower than the unstressed case.

Table 1 Frequency parameter Ω for different R/t ratios

R/t \ $n\rightarrow$	1	2	3	4	5	6
			$\Omega\times10^2$; $\bar{q}=0.00002$			
40.0	0.3467	0.3062	0.2392	0.2493	0.3173	0.4192
66.67	0.2066	0.1839	0.1446	0.1497	0.1716	0.2123
80.0	0.1720	0.1539	0.1228	0.1283	0.1491	0.1777
100.0	0.1387	0.1240	0.1014	0.1048	0.1232	0.1504
133.33	0.1032	0.0945	0.0804	0.0861	0.1022	0.1222
				$\bar{q}=0.0$		
40.0	0.3460	0.3003	0.2182	0.2100	0.2652	0.3618
66.67	0.2056	0.1776	0.1228	0.0989	0.1060	0.1346
80.0	0.1707	0.1474	0.1011	0.0779	0.0780	0.0955
100.0	0.1370	0.1174	0.07996	0.0592	0.0549	0.0633
133.33	0.1016	0.0900	0.0593	0.0425	0.0361	0.03826

Table 1 gives the frequency parameter Ω for various values of R/t and n. It is found that Ω increases as R/t decreases for all values of n and the difference in the fundamental frequencies of stressed and unstressed cases becomes larger as R/t increases. It appears that the number of waves (n) into which a shell deforms at the fundamental frequency is insensitive to R/t ratio for a stressed shell.

It is common practice to consider the effect of membrane forces of intial state alone on the vibration behavior of a shell. An attempt is made to find the effect of meridional rotations, caused by the internal pressure, on the fundamental frequency of stressed shells. The initial rotation has significant effect on the frequencies of short shells. As an example, for a torispherical shell with $L_c/R=0.5$ and $\bar{q}=0.00002$, the parameter Ω, without initial rotation is 0.211×10^{-2} and with the rotation is 0.2649×10^{-2}. No change in the wave number, n ($=6$) is observed for the cases considered.

References

[1] Fung, Y. C., Sechler, E. E., and Kaplan, A., "On the Vibration of Thin Cylindrical Shells under Internal Pressure," *Journal of the Aeronautical Sciences*, Vol. 24, 1957, pp. 650-660.

[2] Armenakas, A. E., "Influence of Initial Stress on the Vibrations of Simply Supported Circular Cylindrical Shells," *AIAA Journal*, Vol. 2, July 1964, pp. 1607-1612.

[3] Dym, C. L., "Vibrations of Pressurized Orthotropic Cylindrical Membranes," *AIAA Journal*, Vol. 8, April 1970, pp. 693-699.

[4] Ebner, A. M., "Effects of Prestress on the Vibration Behavior of Certain Shells of Revolution," *AIAA Journal*, Vol. 9, April 1971, pp. 742-743.

[5] Yang, T. V. and Kim, H. W., "Vibrations and Buckling of Shells under Initial Stress," *AIAA Journal*, Vol. 11, Nov. 1973, pp. 1525-1531.

[6] Sanders, J. L., "Nonlinear Theories for Thin Shells," *Quarterly of Applied Mathematics*, Vol. 21, 1963, pp. 21-36.

[7] Goldberg, J. E., Setlur, A. V., and Alspaugh, D. W., "Computer Analysis of Noncircular Cylindrical Shells," presented at the Conference of the International Association for Shell Structures, Budapest, Hungary, 1965.